

# CORRELATION OF METALLIC THERMAL AND ELECTRICAL CONDUCTIVITIES FOR BOTH SOLID AND LIQUID PHASES

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**Abstract**—This paper is intended to help with the derivation of thermal conductivity,  $\lambda$ , from a knowledge of electrical conductivity,  $\sigma$ .

New data are presented for several copper alloys and molten aluminium, and the information available for the solid and liquid phases of several electrical conductors is reviewed.

For specified temperature ranges, mainly above normal, simple correlating equations of the type first proposed by Smith and Palmer [1] are shown to hold, where  $\lambda = LT\sigma + C$ . With values for the constants  $L$  and  $C$  that are appropriate to the various groups of metals and alloys,  $\lambda$  can often be predicted to within 5–10 per cent, an order of accuracy sufficient for many practical purposes.

Special treatments, are required for materials such as beryllium, chromium and graphite, for which the conduction of heat by phonons is large. Further work seems necessary for these metals and their alloys, also for alloys of tungsten and cobalt and for molten copper. The measurements on molten copper are needed since the values so far obtained are grouped some 13–37 per cent below the extrapolation of the line  $\lambda = 2.32 \times 10^{-8} \sigma T + 0.012$ , which is fitted to within 6 per cent by the data now presented for aluminium and available for several molten metals and alloys of lower  $\lambda$ .

## NOMENCLATURE

$T$ ,	the absolute temperature;
$R$ ,	the gas constant;
$e$ ,	the electronic charge;
$c_p$ ,	the specific heat;
$d$ ,	the density;
$M$ ,	the average molecular weight for alloys;
$\lambda$ ,	the total thermal conductivity;
$\lambda_e$ ,	the electronic component of thermal conductivity;
$\lambda_g$ ,	the lattice component of thermal conductivity;
$\lambda_0$ ,	the impurity scattering component of thermal conductivity;
$W_0$ ,	the impurity scattering component of thermal resistivity;
$\sigma$ ,	the electrical conductivity;
$\rho$ ,	the electrical resistivity;
$\rho_0$ ,	the residual electrical resistivity;
$L_0$ ,	the theoretical Lorenz function;
$L$ ,	the calculated Lorenz function.

## INTRODUCTION

RECENT gatherings of thermal conductivity workers on both sides of the Atlantic have served to emphasize the wide range of technological

problems in which this physical property is an important parameter, and to underline the ever-increasing need for reliable thermal conductivity data. Despite the attention given to this subject, there are but few thermal conductivity apparatus readily available and the determination of this quantity for the wide range of required temperatures and materials has usually to be undertaken by well trained staff in specially equipped laboratories. The measurement of thermal conductivity,  $\lambda$ , is far more difficult than a measurement of electrical conductivity,  $\sigma$ , and it is most important to know if and when  $\lambda$  can be derived from this simpler measurement.

The National Physical Laboratory has been making measurements of  $\lambda$  for over forty years, and, whenever possible, determinations of  $\sigma$  have been included. The purpose of this paper is to show the extent to which, on the basis of these results and those of other workers,  $\lambda$  can be predicted from  $\sigma$  with sufficient accuracy for many practical purposes.

## RELATION BETWEEN THERMAL AND ELECTRICAL CONDUCTIVITY

### Historical

Achard [2] seems to have been the first to direct attention to a proportional relationship

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between  $\lambda$ , and  $\sigma$ , whilst most students are familiar with the work of Wiedemann and Franz [3] and of Lorenz [4] according to which the quantity  $\lambda/\sigma T$ , the so-called Lorenz function, is constant.

Drude [5], on the assumption that free electrons are entirely responsible for both forms of conduction, showed that

$$L_0^* = 3 R^2/e^2 = 2.235 \times 10^{-8} \quad (1)$$

where  $R$  is the gas constant and  $e$  the electronic charge. A later analysis by Sommerfeld [6] led to the numerical coefficient being replaced by  $\pi^2/3$ , giving  $L_0 = 2.45 \times 10^{-8}$ .

Subsequent experimental work at sub-normal temperatures has indicated large departures from this value, except at very low temperatures where the scattering of electrons by impurities gives

rise to the constant residual electrical resistivity,  $\rho_0$ . The thermal resistivity,  $W_0$  arising from this same cause, does obey the relation

$$\rho_0/W_0T = 2.45 \times 10^{-8} \quad (2)$$

For this reason the following account deals mainly with temperatures at or above normal, so any resulting correlations should only be extended below the indicated temperature range after confirmatory tests have been made.

Figure 1, was prepared by the writer for a paper by Schofield [7] and shows that at  $0^\circ\text{C}$   $L$  remains relatively constant at just below the theoretical value for metals and alloys when  $\lambda$  is in excess of about 1, but for lower values of  $\lambda$ ,  $L$  increases at an increasing rate and is often much in excess of the theoretical value.

Figure 2, based on subsequent measurements,

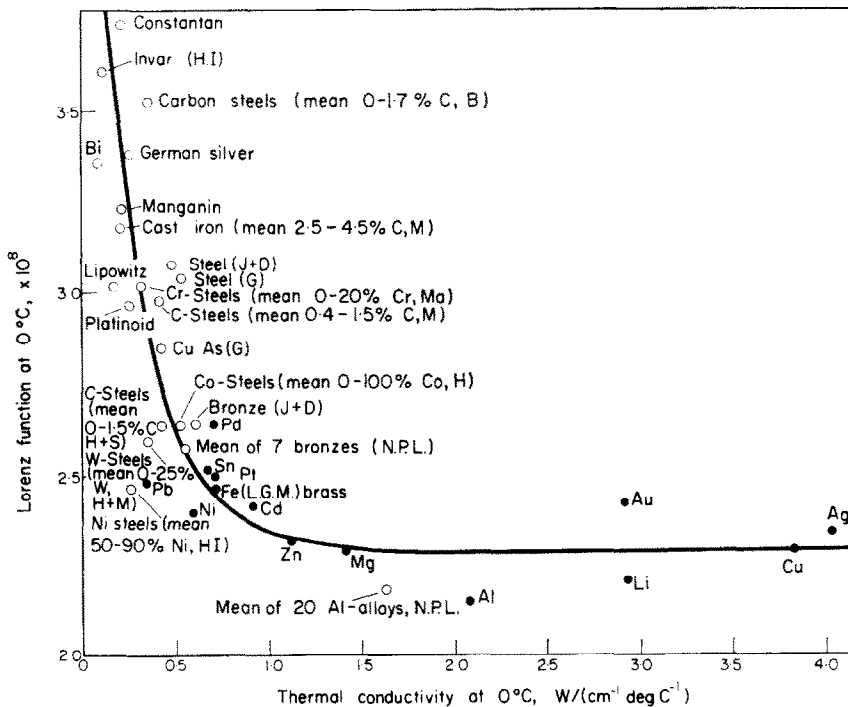


FIG. 1. Relation of Lorenz function and thermal conductivity at  $0^\circ\text{C}$ .

Note: The authorities are as follows: B = Benedicks, 1926; G = Gruneisen, 1900; H = Honda, 1919; H + M = Honda and Matsushita, 1919; H + S = Honda and Simidu, 1917; I = Ingersoll, 1920; J + D = Jaeger and Diesselhorst, 1900; L = Lorenz, 1881; M = Masumoto, 1927; Ma = Matsushita, 1920; N.P.L. = National Physical Laboratory, 1928. J + D were also responsible for determinations on Ag, Cu, Au, Zn, Cd, Ni, Pd, Pb, Constantan, Manganin; Lees (1908) for Cu, Al, Zn, Cd, Ni, Sn, Pb, Brass, Lipowitz Metal, Platinoid, Manganin, German Silver; Gehlhoff and Neumeier (1913) for Bi; Meissner (1920) for Li; N.P.L. (1925) for Mg.

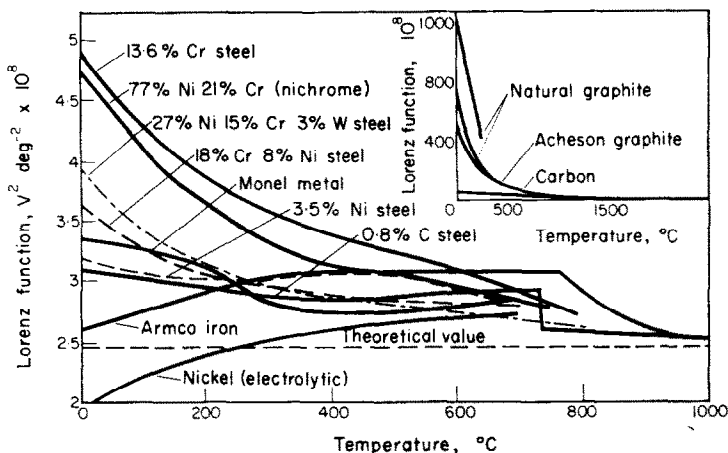


FIG. 2. Variation of Lorenz function with temperature

(Griffiths, Powell and Hickman [8]) serves to show the wide range of values for  $L$  of metals and alloys at normal temperature and the manner in which the values decrease and converge to be within about 10 per cent of the theoretical value at 800°C. The inset portion of this figure relates to carbon and graphite.

It was Königsberger [9] who first pointed out that values of  $L$  in excess of the theoretical value could be expected. In electrical insulators heat is conducted not by electrons but by the lattice and a similar lattice (or phonon) component, may still be present in electrical conductors. In such a case, since  $\lambda = \lambda_e + \lambda_g$  and the WFL law only applies to  $\lambda_e$ , high values of  $L$  could be attributed to the additional component  $\lambda_g$ . In carbons and graphite,  $\lambda_g$  is considerably in excess of  $\lambda_e$ .

### EXPERIMENTAL

#### (a) Copper and its alloys

A notable series of experiments bearing on the prediction of  $\lambda$  from observations of  $\sigma$  or  $\rho$  was due to Smith and Palmer [1]. They reported measurements of  $\lambda$  and  $\sigma$  at 20 and 200°C for no less than 84 copper alloys and showed that up to a value for  $\lambda$  of about 3 the individual points conformed well to a straight line of the form

$$\lambda = L \sigma T + C \quad (3)$$

with  $L = 2.39 + 10^{-8}$  and  $C = 0.075$ . In this

equation  $C$  is regarded as the phonon component  $\lambda_g$ . Thus  $\lambda_g$  is assumed to be constant, and to have the same value for the whole range of samples, whereas there are strong grounds for expecting it to vary as  $1/T$ . The authors state that this equation can be used to predict  $\lambda$  of a copper alloy to an accuracy of at least 0.02 cal  $\text{cm}^{-1} \text{s}^{-1} \text{deg}^{-1}$  (about 0.08  $\text{W cm}^{-1} \text{deg}^{-1}$ ). Furthermore, they state that the results of other workers for pure metals as well as alloys often conform more closely to their equation than to the Wiedemann–Franz–Lorenz relationship. Aluminium is, however, noted as an exception and they anticipate that further investigation will lead to rather different equations being devised for aluminium and probably for other alloy groups.

The National Physical Laboratory has had occasion to make measurements on several specimens of copper and its alloys. These results, which have not previously been published, are set out in Table 1 where comparison is made between the observed values for  $\lambda$  and those calculated from the Smith–Palmer equation. The close agreement confirms that predictions made by means of this equation should suffice for most practical purposes.

Some measurements by Gupta and Banerjee [10] are presented in Table 2. These relate to copper–manganese alloys and are compared both with Smith and Palmer values for similar alloys and with values calculated from their equation.

Table 1. Thermal conductivity of copper and its alloys  
 NPL measurements compared with values calculated from the Smith-Palmer equation

Metal or alloy	$T$	$10^6 \rho$ (ohm cm)	$\lambda$ (W cm <sup>-1</sup> deg <sup>-1</sup> )		
			Measured	Calculated	Difference (%)
98.9% Cu, 0.8% Cr	293	2.03	3.42	3.42*	0.0
Commercial Cu	343	4.42	1.94	1.93	0.3
Commercial Cu	373	4.62	2.03	2.01	1.0
Arsenical Cu, 0.34% As	323	4.39	1.86	1.83 <sub>5</sub>	1.4
99.57% Cu					
Arsenical Cu, 0.34% As	423	4.91	2.09	2.13 <sub>5</sub>	-2.2
99.57% Cu					
Arsenical copper	323	5.1	1.60	1.59	0.6
Arsenical copper	423	5.8	1.80	1.82	-1.1
Arsenical copper	523	6.5	1.99	2.00	-0.5
Arsenical copper	623	7.1 <sub>5</sub>	2.18	2.16	0.9
Cu + 2-3.5% Ni + 0.4-0.8% Si	193	3.24	1.47	1.50	-2.0
Cu + 2-3.5% Ni + 0.4-0.8% Si	273	3.78	1.76	1.80	-2.2
Cu + 2-3.5% Ni + 0.4-0.8% Si	373	4.44	2.03	2.08 <sub>5</sub>	-2.6
Cu + 2-3.5% Ni + 0.4-0.8% Si	473	5.12	2.23	2.28 <sub>5</sub>	-2.4
Cu + 2-3.5% Ni + 0.4-0.8% Si	573	6.51	2.42	2.44 <sub>5</sub>	-1.0
Brass	323	6.51	1.25 <sub>5</sub>	1.26	-0.4
Brass	423	7.45	1.42	1.43 <sub>3</sub>	-0.2
Aluminium brass	323	8.51	0.98 <sub>5</sub>	0.98	0.5
Aluminium brass	423	9.54	1.45 <sub>5</sub>	1.13 <sub>5</sub>	0.9
Phosphor bronze	323	14.9	0.60	0.59 <sub>3</sub>	1.0
Phosphor bronze	373	15.4	0.65 <sub>5</sub>	0.65 <sub>5</sub>	0.0
Phosphor bronze	473	16.5	0.75 <sub>5</sub>	0.76	-0.7
Tin (12%) bronze	323	15.8 <sub>5</sub>	0.53 <sub>5</sub>	0.56	-4.5
Tin (12%) bronze	423	16.9	0.65	0.67 <sub>3</sub>	-3.4
90% Cu 10% Ni	323	19.5	0.47 <sub>5</sub>	0.47	1.1
90% Cu 10% Ni	423	20.4 <sub>5</sub>	0.57	0.57	0.0
Nominally 70% Cu 30% Ni	323	40.1	0.26 <sub>5</sub>	0.26 <sub>7</sub>	-0.8
Nominally 70% Cu 30% Ni	423	40.7 <sub>5</sub>	0.31	0.32 <sub>3</sub>	-4.0
66% Cu, 30% Ni 2% Fe, 2% Mn	323	45.5	0.23	0.24	-4.2
66% Cu, 30% Ni 2% Fe, 2% Mn	423	46.4	0.27 <sub>5</sub>	0.29 <sub>5</sub>	-6.2

\* From Table of Appendix B, Smith and Palmer [1].

These two sets of experimental data are seen to be in good agreement and to confirm that larger differences of up to some 15 per cent are to be expected when the Smith-Palmer equation is used for copper alloyed with about 20 per cent of manganese.

#### (b) Metals

Figure 3 relates to the suggestion that the Smith-Palmer equation also holds for many metallic elements and shows  $\lambda$  plotted against  $\sigma T$  for 24 of these that have been studied at the National Physical Laboratory, often over wide

Table 2. Copper-manganese alloys: comparison of data by Gupta and Banerjee with calculated values and with original Smith-Palmer values

Mn (%)	$\lambda$ at 293°K due to Smith and Palmer			$\lambda$ at 323°K due to Gupta and Banerjee		
	Measured	Calculated	Difference (%)	Measured	Calculated	Difference (%)
0.43	2.26	2.14	5.5	—	—	—
1.05	1.50 <sub>5</sub>	1.41	6.5	—	—	—
1.77	1.02	0.95	7	—	—	—
4.55	0.49	0.46 <sub>5</sub>	5	—	—	—
4.94	—	—	—	0.46	0.43 <sub>5</sub>	5.5
9.53	0.26	0.27 <sub>2</sub>	-4.5	—	—	—
10.64	—	—	—	0.25	0.28	-11
19.82	0.15	0.17 <sub>7</sub>	-15	—	—	—
21.38	—	—	—	0.15	0.17 <sub>6</sub>	-15
24.50	—	—	—	0.13 <sub>8</sub>	0.16 <sub>8</sub>	-15.5
29.15	—	—	—	0.13 <sub>4</sub>	0.14 <sub>6</sub>	-8
40.21	—	—	—	0.12 <sub>5</sub>	0.13	-4

Table 3. Comparison of reference-table data at 293°K (Metals Handbook, 1961) with values calculated from the Smith-Palmer equation and with recent NPL values

Metal	Metals Handbook 1961		Calculated ( $\lambda$ )	Difference, % of calculated	NPL values	
	$10^6 \rho$	$\lambda$			$\lambda$	Difference, % of calculated
Cobalt	6.24	0.69	1.20	-42	0.96 [13]	-20
Indium	8.27	0.24	0.91	-77	0.86 [11]	-5.5
Iridium	5.3	0.59	1.39 <sub>5</sub>	-58	1.46 [14]	4.7
Rhenium	19.3	0.71	0.44	62	0.48 [15]	9.7
Rhodium	4.51	0.88	1.63	-46	1.51 [14]	-7.4

ranges of temperature. For only 10 of the metals studied do the experimental values for  $\lambda$  differ from the predictions of the Smith-Palmer equation by more than 10 per cent. The greatest departures are for bismuth, chromium and tungsten.

Table 3 lists some metals which were investigated because of the considerable differences indicated by reference-table data and shows the marked improvement that has resulted. The greatest difference is for indium for which Powell, Woodman and Tye [11] find  $\lambda$  to be increased by a factor of no less than 3.6. Indeed, the suggestion has since been made [12] that a value for  $c_p$  had at some stage been tabulated in mistake for  $\lambda$  and that this error had been perpetuated by subsequent compilers of tables.

Table 3 emphasizes the value of making independent checks of this kind before any new values for  $\lambda$  are used for design purposes.

(c) *Other groups of metals and alloys*

Further consideration will now be given to the suggestion that the constants of the equation might well be modified for other groups of alloys. Indeed, Grootenhuis, Powell and Tye [16] showed this to be desirable for another type of copper alloy. Figure 4 shows  $\lambda$  plotted against  $\sigma T$  using data obtained by these workers over the range 50 to 200°C for several porous bronze samples prepared from spherical particles by powder metallurgy methods. These results conform to the line

$$\lambda = 2.43 \times 10^{-8} \sigma T + 0.021 \quad (4)$$

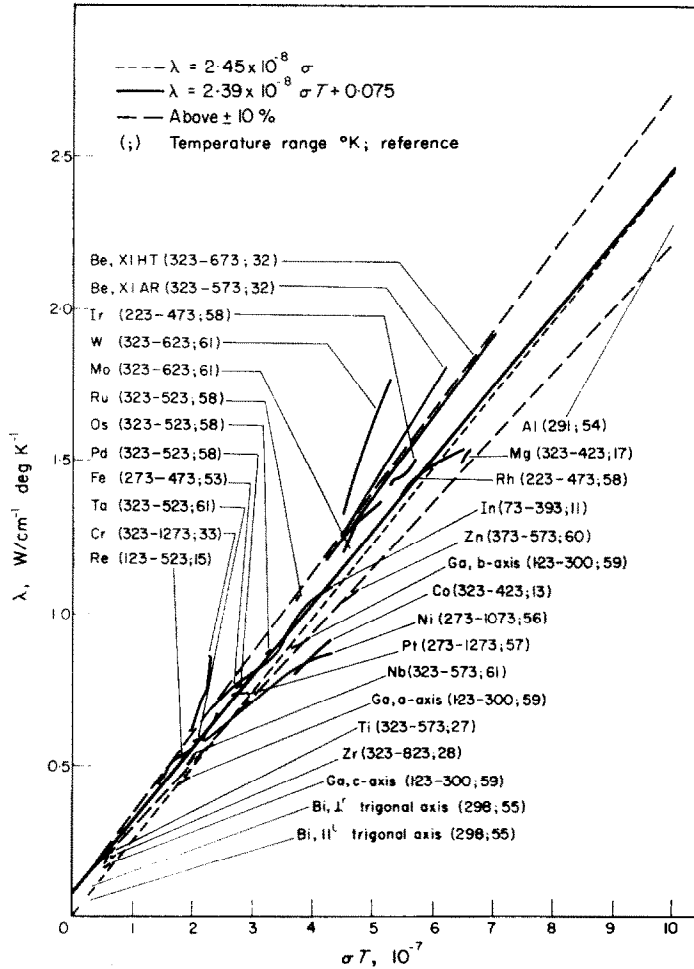


FIG. 3. N.P.L. data for metallic elements: Comparison with Smith-Palmer equation.

which is approximately parallel to, but some 29–36 per cent below, the Smith-Palmer line. The smaller intercept is consistent with the smaller value of  $\lambda_0$  that is to be expected for the more subdivided sintered material.

Figure 5 is a similar plot for 2 samples of magnesium and 28 magnesium alloys [17] which support Powell [18] and fit a line of lower slope and greater intercept.

$$\lambda = 2.206 \times 10^{-8} \sigma T + 0.096 \quad (5)$$

For this wide range of magnesium alloys the majority of the experimental values agree with this line to within 3 per cent and all agree to within 6 per cent.

The constants and other details of similar equations that have been proposed for various alloy groups are given in Table 4. These lines are plotted in Fig. 6, where the length of each line indicates the particular range of values for which it has been derived.

The figure is helpful in showing that use of the appropriate equation is essential for materials where  $\lambda$  values are low. Alloys of magnesium and aluminium could be treated as one group satisfying the equation

$$\lambda = 2.21 \times 10^{-8} \sigma T + 0.10 \quad (6)$$

Further work is still required to investigate

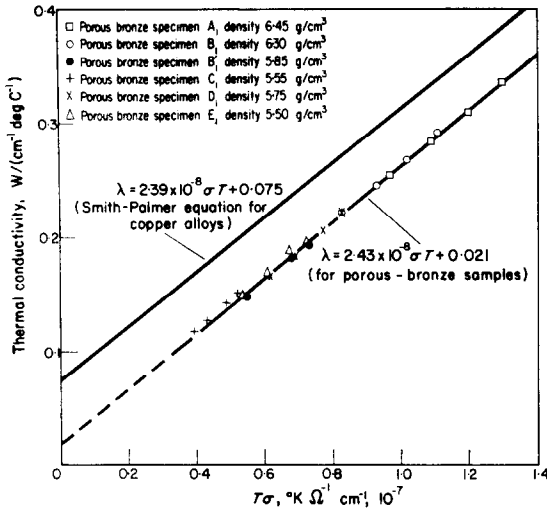


FIG. 4. Porous bronze: thermal conductivity as a function of the product of absolute temperature and electrical conductivity.

alloys of some of the more exceptional metals, such as beryllium, cobalt and tungsten.

Ewing, Walker, Grand and Miller [29] have used the experimental data for 140 metals and alloys to derive what is considered to be a more

general single equation

$$\lambda = 2.61 \times 10^{-8} \sigma T(1 - 7.7 \times 10^{-10} \sigma T/c_p d) + 97 c_p d^2/MT \quad (7)$$

The overall deviation for all the metals studied is stated to be  $\pm 5$  per cent and they conclude that  $\lambda$  for any metal or alloy other than a semiconductor can normally be estimated to within 5–10 per cent so long as reliable values for  $\sigma$  are available. This suggests that the reliability of the Ewing equation is comparable with that of the appropriate two-term equation. The second term of equation (7) becomes most effective for the good conducting metals, which are seen from Fig. 1 to have  $L$  below the theoretical value. For copper at room temperature the first two terms yield  $L$  of about  $2.2 \times 10^{-8}$ . The third term represents  $\lambda_g$ , and is now seen to vary as  $T^{-1}$ .

Bungardt and Kallenbach [30, 31], on the other hand, have recommended for aluminium and magnesium and their alloys equations of the form

$$\lambda/T = L \sigma + C \quad (8)$$

the constants  $L$  and  $C$  being  $2.05 \times 10^{-8}$  and

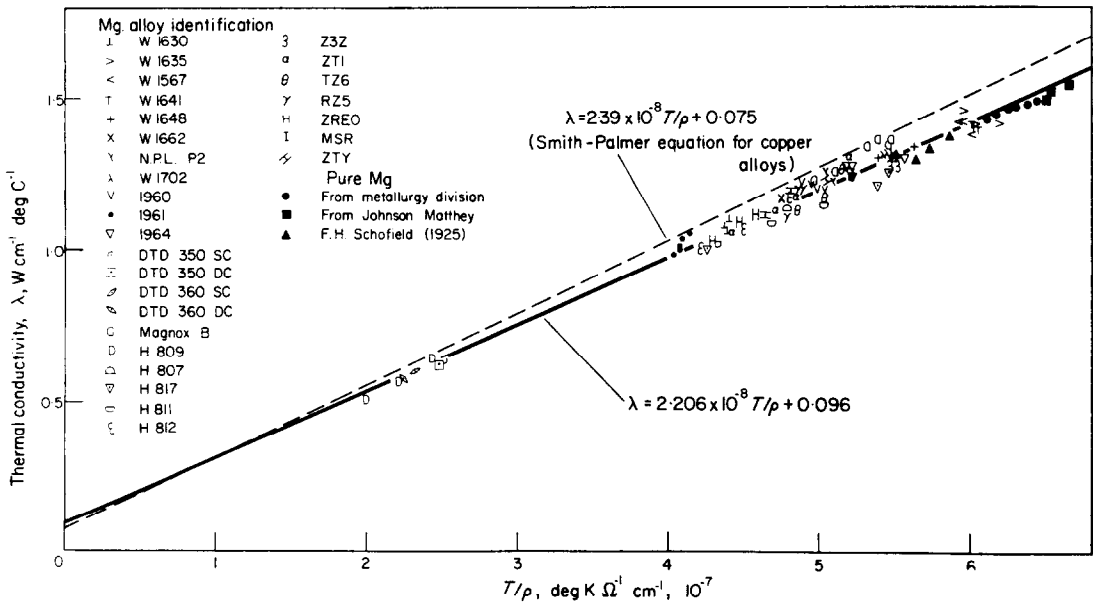


FIG. 5. Magnesium and magnesium alloys: Thermal conductivity,  $\lambda$  plotted against  $10^{-7}$  times absolute temperature divided by electrical resistivity,  $10^{-7} T/\rho$ .

Table 4. Constants of equations appropriate to various metals and their alloys

Metal group	$10^8 L$	$C$	Range		Accuracy % (approx.)	Reference
			(°K)	$(10^{-7} \sigma T)$		
Al alloys	2.10	0.126	293–473	4.7–9.5	6*	[19]
Al alloys	2.22	0.105	273–573	3.2–8.3	8*	[20]
Cu alloys	2.39	0.075	293–473	0.4–17.3	10	[1]
Cu alloy (porous bronze)	2.43	0.021	293–473	0.4–1.3	5	[16]
Fe and steels	2.62	0.025	113–1173	0.3–3.1	12†	[21]
$\alpha$ -Fe and ferrous steels	2.43	0.092	373–1173	0.6–3.0	10	[22]
$\gamma$ -Fe and austenitic steels	2.39	0.042	373–1173	0.3–1.0	10	[22]
Fe–Ni alloys	2.92	0.030	123–813	0.1–1.7	10‡	[23]
Mg and Mg alloys	2.16	0.092	323–523	1.5–7.0	6	[18]
Mg and Mg alloys	2.206	0.096	318–773	1.5–7.0	6	[17]
Ni and Ni alloys	2.13	0.084	148–1273	0.25–3.9	20	[24]
Ni–Cr alloys (Nimonic type)	2.20	0.060	323–1123	0.24–0.95	5	[25]
Ti and 6 Ti alloys	2.62	0.0208	311–811	0.2–0.6	10	[26]
Ti alloys	2.39	0.0292	323–723	0.19–0.42	7	[27]
Zr alloys	2.503	0.0223	323–523	0.2–0.6	10	[28]

\* Measured values for alloys containing Si give larger positive differences.

† Many cast irons and some steels, e.g. 13% Cr-steel below 673°K, give larger positive differences.

‡ Greater differences were given by an alloy with nearly 80% Ni, for which a larger value of  $C$  is to be expected.

$5.02 \times 10^{-5}$  for aluminium and  $2.26 \times 10^{-8}$  and  $1.675 \times 10^{-4}$  for magnesium. This procedure makes  $\lambda_g$  proportional to  $T$ .

#### (d) Some exceptional materials

The special methods adopted when treating the metals beryllium [32] and chromium [33], which proved to give exceptionally large and variable values for  $\lambda_g$ , are indicated below. For each metal these measurements have been made on relatively few samples, so further measurements are desirable. Information on graphite is also included in this section.

**Beryllium.** Measurements made on five samples of beryllium before and after heat treatment indicated values of  $\lambda_g$  approaching 50 per cent at 323°K, but these decreased with increase in  $T$  and the results for the range 323–1000°K all agreed to within 6 per cent with the equation

$$\lambda = 10^{-8} T \sigma \left( 2.57 - \frac{249}{T} \right) + \frac{268}{T} - 0.151 \quad (9)$$

**Chromium.** Only one electrodeposited sample has been studied, for which both  $\lambda$  and  $\sigma$  has been found to increase after heat treatment at

successively higher temperatures. Following heat treatment at 1683°K,  $\lambda$  at 323°K was 0.86, some 3.65 times the value for the deposited metal, whilst the corresponding derived values for  $\lambda_g$  were 0.31 compared with an initial value of 0.07. The deposited metal contains gaseous impurities, and it is to be expected that both  $\lambda$  and  $\lambda_g$  will increase as these are removed and the density increases, but chromium seems to be a metal for which  $\lambda_g$  amounts to more than 30 per cent of  $\lambda$  and the examination of further samples would clearly be of interest.

**Graphite.** The inclusion of graphite in this account must seem rather surprising. Having a value of  $L$  some 150–450 times the theoretical value,  $\lambda_g$  must predominate and any useful correlation of  $\lambda$  and  $\sigma$  would appear to be unlikely. However, several have been proposed and Mason and Knibbs [34] consider that it is because the flow of both heat and electricity is mainly restricted to the crystal layer planes and controlled by scattering at the crystal boundaries that this is possible.

Powell [35] showed that for a particular sample of Acheson graphite the following equation



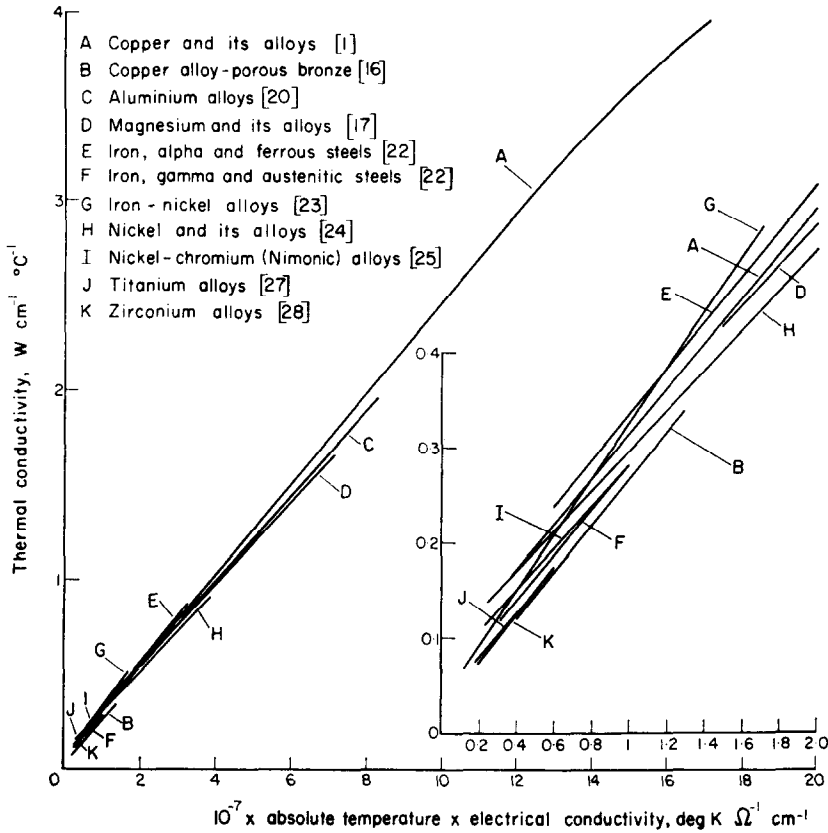


FIG. 6. Relation of thermal conductivity and product of absolute temperature and electrical conductivity.

$$\lambda = 0.123 T^{0.8} \sigma \quad (10)$$

held from room temperature to 1073°K.

Powell and Schofield [36] found that above about 1500°K the predicted values erred on the high side and by 2800°K exceeded the experimental values by some 60 per cent. A criticism of this work is that whereas  $\sigma$  was measured longitudinally,  $\lambda$  was measured radially and the material was considered to be isotropic. This now seems rather unlikely.

Powell [37] reported measurements from 323 to 573°K for several graphite samples and found that for five samples cut from a block of Ceylon graphite, two samples of Acheson graphite and one of Hilger "H.S." graphite

$$\lambda = 2.22 T^{-1.3} \sigma + 0.184 \quad (11)$$

whilst for two samples of Cumberland graphite

$$\lambda = 3.1 T^{-1.3} \sigma + 0.25 \quad (12)$$

Mason and Knibbs refer to several similar relationships found to apply to various grades of reactor and other graphites and it is clearly evident that even for graphites much useful information relating to  $\lambda$  can be derived from a knowledge of  $\sigma$ .

(e) Other materials having high Lorenz functions

Cast irons have already been mentioned as materials known to have unusually high values for  $L$ . Work is in progress on materials of this type in the hope of understanding them more fully and maybe of allowing predictions to be made. Results for zirconium and titanium diborides, titanium carbide and for mixtures of  $TiB_2$  and  $TiC$  have been reported [38]. These compounds have high values of  $\lambda_0$  for which the temperature dependence varies from compound to compound.

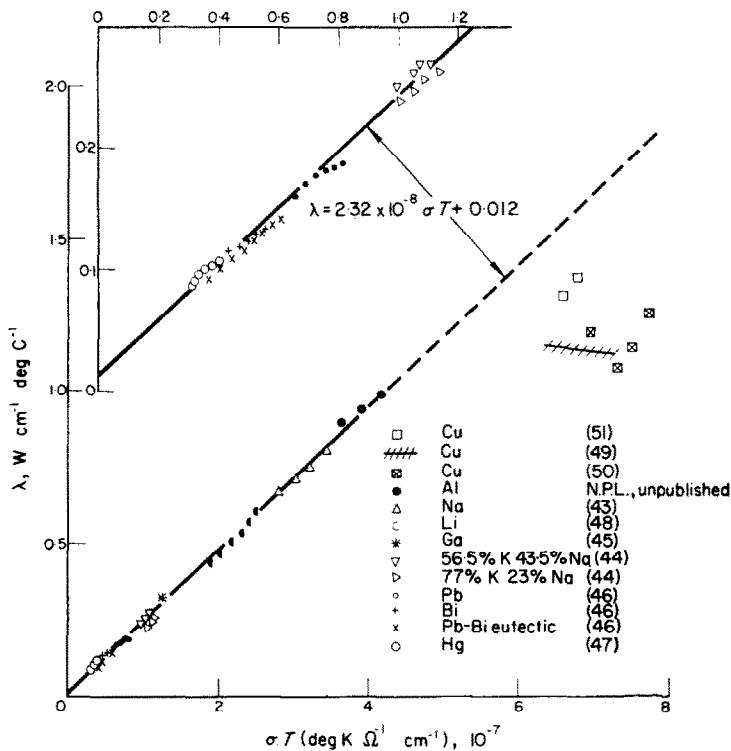


FIG. 7. Liquid metals: Thermal conductivity against the product of electrical conductivity and absolute temperature.

(f) *Liquid phase*

A survey of existing data for  $\lambda$  liquid metals was attempted [39] when their use as liquid coolants was under consideration. Despite a fair degree of uncertainty in some of the experimental data it was clear that for liquid metals,  $L$  had values of about the theoretical order. Some indication of these uncertainties is given by the three sets of measurements available for  $\lambda$  for aluminium in the liquid phase [40–42] which covered just about a two-fold range of values and included both positive and negative temperature coefficients.

With a view to resolving these differences observations have recently been completed on a sample of S.P. aluminium and are in progress for an aluminium alloy. For aluminium,  $\lambda$  is found to increase linearly from 0.90 at 973°K to 0.98 at 1273°K, whilst  $L$  decreases from  $2.43 \times 10^{-8}$  to  $2.40 \times 10^{-8}$ .

These values for  $L$  are in fair accord with those

derived from other recent measurements on molten metals due to Ewing *et al.* [43, 44], Briggs [45], Powell and Tye [46, 47] and Cooke [48]. In Fig. 7 these data have been used for a plot of  $\lambda$  against  $\sigma T$ . Values due to four different sets of workers for seven metals and three alloys are seen to conform well to the straight line.

$$\lambda = 2.32 \times 10^{-8} \sigma T + 0.012 \quad (13)$$

The maximum departure of any of the points from this line is 6 per cent, which is a surprising result when compared with the degree of scatter that was observed when earlier measurements were examined in a similar manner.

One is tempted to conclude that for molten metals or alloys  $\lambda$  can be derived with reasonable certainty by the use of equation (13), and there are good grounds for believing that this conclusion will receive support as additional results become available. Further work on molten copper is certainly necessary for points derived

from the published values for  $\lambda$  due to Fieldhouse, Hedge, Lang and Waterman [49], McClelland, Rasor, Dahleen and Zehms [50] and Lucks and Deem [51], using  $\sigma T$  values extrapolated from the measurements of Roll and Metz [52], are seen to be grouped some 13–37 per cent below the extension of the line fitting the other experimental points of Fig. 7.

### CONCLUSIONS

Use of the foregoing equations should enable  $\lambda$  to be derived from a knowledge of  $\sigma$  or  $\rho$  with an accuracy of about 5–10 per cent, which should suffice for many purposes. Metals and alloys on which further work is desirable include bismuth, cobalt, chromium, tungsten, cast irons and molten copper.

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### REFERENCES

1. C. S. SMITH and E. W. PALMER, Thermal and electrical conductivities of copper alloys, *Trans. Amer. Inst. Min. Metall. Engrs* **117**, 225–243 (1935).
2. F. C. ACHARD, *Mem. Acad. Berlin*, 27 (1779).
3. G. WIEDEMANN and R. FRANZ, On the thermal conductivity of metals, *Ann. Physik* **89**, 497–531 (1853).
4. L. LORENZ, Determination of the degree of heat in absolute measure *Ann. Physik* **147**, 429 (1872).
5. P. DRUDE, On the electronic theory of metals, *Ann. Phys.* **1**, (4) 566–613 (1900).
6. A. SOMMERFELD, *Z. Phys.* **47**, 27 (1928).
7. F. H. SCHOFIELD, The thermal conductivity of metals and its relation to electrical conductivity, *J. Sheffield Engng Metall. Soc.* (1929).
8. E. GRIFFITHS, R. W. POWELL and M. J. HICKMAN, Thermal conductivity of some industrial materials, *J. Inst. Fuel* **15**, 107 (1942).
9. J. KONIGSBERGER, On the relative of thermal to electrical conductivity. *Phys. Z.* **8**, 237 (1907).
10. M. K. GUPTA and T. BANERJEE, Thermal and electrical conductivity of copper–manganese alloys. *Nat. Metall. Lab. Tech. J.* **4**, (4) 12 (1962).
11. R. W. POWELL, MARGARET J. WOODMAN and R. P. TYE, The thermal conductivity and electrical resistivity of indium, *Phil. Mag.* **7**, 1183 (1962).
12. N. M. ROISIN, N. S. MOSTOVLYANSKII and R. K. STROD, Thermal conductivity of indium. *Fiz. Tverdogo Tela* **5**, (4) 1216 (1963).
13. R. W. POWELL, Some preliminary measurements of the thermal conductivity and electrical resistivity of cobalt. *Cobalt* (No. 24) 145–150 (Sept. 1964).
14. R. W. POWELL and R. P. TYE, Thermal conductivity measurements down to  $-190^{\circ}\text{C}$ : iridium and rhodium. *Proc. IXth Int. Congr. Refrig.* **1**, 2083–2087 (1955).
15. R. W. POWELL, R. P. TYE and MARGARET J. WOODMAN, The thermal conductivity and electrical resistivity of rhenium, *J. Less-Common Metals* **5**, 49–56, (1963).
16. P. GROOTENHUIS, R. W. POWELL and R. P. TYE, Thermal and electrical conductivity of porous metals made by powder metallurgy methods, *Proc. Phys. Soc. B* **65**, 502–511 (1952).
17. R. W. POWELL, M. J. HICKMAN and R. P. TYE, The thermal and electrical conductivity of magnesium and some magnesium alloys, *Metallurgia* **70**, 159 (1964).
18. R. W. POWELL, The thermal and electrical conductivities of some magnesium alloys, *Phil. Mag.* **27**, 677–686 (1939).
19. L. W. KEMPF, C. S. SMITH and C. S. TAYLOR, Thermal and electrical conductivities of aluminium alloys, *Trans. Amer. Inst. Min. Metall. Engrs*, **124**, 287–298 (1937).
20. R. W. POWELL and M. J. HICKMAN, Some physical properties of aluminium alloys at elevated temperatures, I, Thermal conductivity and electrical resistivity, *Metallurgia* **41**, 15 (1949).
21. R. W. POWELL, A survey of existing data on the thermal and electrical conductivities of irons and steels, Iron and Steel Institute, Special Report No. 24, 253–268 (1939).
22. R. W. POWELL, Thermal conductivities of some gas-turbine materials, Iron and Steel Institute, Special Report, No. 43, 315–318 (1952).
23. D. R. FLYNN and H. E. ROBINSON, National Bureau of Standards, Private communication (1962).
24. M. E. FINE, Correlation between electrical and thermal conductivity in nickel and nickel alloys, *J. Metals* **188**, 951 (1950).
25. R. W. POWELL and R. P. TYE, Thermal and electrical conductivities of nickel–chromium (Nimonic) alloys, *Engineer, Lond.* **209**, 729–732 (1960).
26. H. W. DEEM, W. D. WOOD and C. F. LUCKS, The relationship between electrical and thermal conductivities of titanium alloys, *Trans Metall. Soc. A.I.M.E.* **212**, 520 (1958).
27. R. W. POWELL and R. P. TYE, Thermal conductivity of titanium and its alloys, *J. Less-Common Metals* **3**, 226–233 (1961).
28. R. W. POWELL and R. P. TYE, The thermal and electrical conductivities of zirconium and of some zirconium alloys, *J. Less-Common Metals* **3**, 202–215 (1961).
29. C. T. EWING, B. E. WALKER, J. A. GRAND and R. R. MILLER, Thermal conductivity of metals, *Liquid Metals Technology—Part 1*, Chemical Engineering Progress Symposium Series, **53**, (20) 19–24 (1957).
30. W. BUNGARDT and R. KALLENBACH, On the relation between the thermal and electrical conductivity of

- aluminium and magnesium alloys. *Metallwiss. Techn.* pp. 317 and 365 (1950).
31. W. BUNGARDT and R. KALLENBACH, On the thermal and electrical conductivity of some aluminium alloys for temperatures up to 400°C, *Z. Metallk.* **42**, 82 (1951).
  32. R. W. POWELL, The thermal and electrical conductivities of beryllium, *Phil. Mag.* **44**, 645-663 (1953).
  33. R. W. POWELL and R. P. TYE, The influence of heat treatment on the electrical resistivity and thermal conductivity of electrodeposited chromium, *J. Inst. Metals* **85**, 185-192 (1957).
  34. I. B. MASON and R. H. KNIBBS, The thermal conductivity of artificial graphites and its relation to electrical resistivity, UKAEA, Research Group Report, *AERE-R.3973* H.M.S.O., London (March 1962).
  35. R. W. POWELL, The thermal and electrical conductivities of a sample of Acheson graphite from 0 to 800°C, *Proc. Phys. Soc.* **49**, 419 (1937).
  36. R. W. POWELL and F. H. SCHOFIELD, The thermal and electrical conductivities of carbon and graphite to high temperatures, *Proc. Phys. Soc.* **51**, 153 (1939).
  37. R. W. POWELL, The thermal conductivities of metallic conductors and their estimation, *Inst. Mech. Engrs, Amer. Soc. Mech. Engrs, Proceedings of the General Discussion on Heat Transfer*, 290-294 (1951).
  38. R. W. POWELL and R. P. TYE, The thermal conductivities of some electrically conducting compounds. Presented to Special Ceramics Conference at British Ceramics Research Association, July (1964).
  39. R. W. POWELL, Thermal conductivities of molten metals and alloys, *J. Iron St. Inst.* **162**, 315-324, (1949).
  40. S. KONNO, On the variation of the thermal conductivity during fusion of metals, *Sci. Rep. Tokoku Imperial University* **8**, 169-179 (1919).
  41. C. C. BIDWELL and C. L. HOGAN, Thermal conductivity of aluminium: solid and liquid states, *J. Appl. Phys.* **18**, 776-779 (1947).
  42. C. L. HOGAN, Bell Aircraft Corp. Report 56-982-010, Oct. 1st 1950. (Referred to in *Liquid Metals Handbook*, Atomic Energy Commission, Department of the Navy, Washington, D.C., June, 1952), also Lehigh University Doctoral Dissertation, 1-43 (1950).
  43. C. T. EWING, J. A. GRAND and R. R. MILLER, Thermal conductivity of liquid sodium and potassium, *J. Amer. Chem. Soc.* **74**, 11 (1952).
  44. C. T. EWING, R. E. SEEBOLD, J. A. GRAND and R. R. MILLER, Thermal conductivity of mercury and two sodium-potassium alloys, *J. Phys. Chem.* **59**, 524 (1955).
  45. L. J. BRIGGS, Gallium: thermal conductivity, supercooling, negative pressure, *J. Chem. Phys.* **26**, 784-786 (1957).
  46. R. W. POWELL and R. P. TYE, Experimental determinations of the thermal and electrical conductivities of molten metals, *Proceedings of the Joint Conference on Thermodynamic and Transport Properties of Fluids*, pp. 182-187. *Inst. Mech. Engrs.* (1958).
  47. R. W. POWELL and R. P. TYE, The thermal and electrical conductivity of liquid mercury, *International Developments in Heat Transfer*, pp. 856-862. A.S.M.E. (1961).
  48. J. W. COOKE, Experimental determination of the thermal conductivity of molten lithium from 320 to 830°C, *J. Chem. Phys.* **40**, 1902 (1964).
  49. I. B. FIELDHOUSE, J. C. HEDGE, J. I. LANG and T. E. WATERMAN, Measurements of thermal properties, *WADC Technical Report 55-495*, Part II 1-18; *AD 110510* (1956).
  50. J. D. McCLELLAND, N. S. RASOR, R. C. DAHLEEN and E. H. ZEHMS, Thermal conductivity of liquid copper, *WADC Technical Report 56-400* Part II 1-6; *AD 118243* (1957).
  51. C. F. LUCKS and H. W. DEEM, Thermal properties of thirteen metals, American Society for Testing Materials Special Technical Publication No. 227, 1-29 (1958).
  52. A. ROLL and H. METZ, Electrical resistivity of molten metals. *Z. Metall.* **48**, 272-280 (1957).
  53. F. ADCOCK and C. A. BRISTOW, Iron of high purity, *Proc. Roy. Soc. A* **153**, 172-200 (1935).
  54. E. GRIFFITHS, The thermal and electrical conductivity of a single crystal of aluminium, *Proc. Roy. Soc. A* **115**, 236 (1927).
  55. G. W. C. KAYE, The thermal and electrical resistance of bismuth single crystals; the effects of temperature and magnetic fields, *Proc. Roy. Soc. A* **170**, 561 (1939).
  56. R. W. POWELL, Thermal conductivity of solid materials at high temperatures, *Research* **7**, 492 (1954).
  57. R. W. POWELL and R. P. TYE, The promise of platinum as a high-temperature thermal conductivity reference material, *Brit. J. Appl. Phys.* **14**, 662 (1963).
  58. R. W. POWELL, R. P. TYE and MARGARET J. WOODMAN, Thermal conductivities and electrical resistivities of the platinum metals, *Plat. Met. Rev.* **6**, 138 (1962).
  59. R. W. POWELL, MARGARET J. WOODMAN and R. P. TYE, Further measurements relating to the anisotropic thermal conductivity of gallium, *Brit. J. Appl. Phys.* **14**, 432 (1963).
  60. F. H. SCHOFIELD, The thermal and electrical conductivities of some pure metals, *Proc. Roy. Soc. A* **107**, 206 (1925).
  61. R. P. TYE, Preliminary measurements on the thermal and electrical conductivities of molybdenum, niobium, tantalum and tungsten, *J. Less-Common Metals* **3**, 13 (1961).

**Résumé**—Cet article a pour but d'aider à obtenir la conductivité thermique,  $\lambda$ , connaissant la conductivité électrique,  $\sigma$ .

De nouveaux résultats sont présentés pour plusieurs alliages de cuivre et l'aluminium fondu, et l'information disponible sur les phases solides et liquides de plusieurs conducteurs électriques est passée en revue.

Dans des gammes spécifiées de température, principalement au-dessus de la normale, on montre

que des équations simples de corrélation, du type  $\lambda = LT\sigma + C$  proposé d'abord par Smith et Palmer [1] sont valables. Avec des valeurs pour les constantes  $L$  et  $C$  qui sont appropriées aux différents groupes de métaux et d'alliages,  $\lambda$  peut souvent être prédit avec une erreur de 5 à 10 pour cent, précision suffisante pour beaucoup de buts pratiques.

On a besoin de méthodes spéciales de traitement pour des matériaux tels que le béryllium, le chrome et le graphite, pour lesquels la conduction de la chaleur par phonons est élevée. Une étude ultérieure semble nécessaire pour ces métaux et leurs alliages, ainsi que pour les alliages de tungstène et de cobalt et pour le cuivre fondu. Les mesures sur le cuivre fondu sont nécessaires, car les valeurs obtenues jusqu'à présent sont groupées de 13 à 37 pour cent au-dessous de l'extrapolation de la ligne  $\lambda = 2,32 \cdot 10^{-8} \sigma T + 0,012$  qui est vérifiée à moins de 6 pour cent par les résultats présentés actuellement pour l'aluminium et disponibles pour plusieurs métaux et alliages fondus de conductivité thermique  $\lambda$  plus faible.

**Zusammenfassung**—Diese Arbeit soll die Ableitung des Wärmeleitvermögens  $\lambda$  aus der bekannten elektrischen Leitfähigkeit  $\sigma$  ermöglichen.

Für mehrere Kupferlegierungen und für Aluminiumschmelze werden neue Werte angegeben und die verfügbaren Unterlagen über die feste und flüssige Phase mehrerer elektrischer Leiter werden überprüft.

Für vorgegebene Temperaturbereiche, die hauptsächlich über dem Normzustand liegen, erweisen sich einfache Beziehungen von dem Typ  $\lambda = LT\sigma + C$ , der zuerst von Smith und Palmer [1] vorgeschlagen wurde, als ausreichend. Mit den Werten für die Konstanten  $L$  und  $C$ , die auf die verschiedenen Gruppen von Metallen und Legierungen zutreffen, kann  $\lambda$  oft bis auf 5 bei 10 Prozent genau vorherbestimmt werden, eine Genauigkeit, die für viele praktische Zwecke genügt. Gesondert müssen Materialien wie Beryllium, Chrom und Graphit, bei denen die Wärmeleitung durch Phononen gross ist, behandelt werden. Für diese Metalle und deren Legierungen wie auch für Legierungen von Wolfram und Kobalt und Kupferschmelze scheint weitere Forschungsarbeit nötig zu sein. Die Messungen an Kupferschmelzen werden benötigt, da die bisher erhaltenen Werte ungefähr 13 bis 37 Prozent unter der Extrapolation der Linie  $\lambda = 2,32 \times 10^{-8} \sigma T + 0,012$  liegen, welche sich innerhalb 6 Prozent den jetzt angegebenen Werten für Aluminium und den verfügbaren Daten für mehrere Metallschmelzen und Legierungen mit niedrigerem  $\lambda$  anpasst.

**Аннотация**—Целью настоящей работы является установление связи между теплопроводностью  $\lambda$  и электропроводностью  $\sigma$ .

Представлены новые данные для нескольких медных сплавов и расплавленного алюминия, и проведен анализ имеющихся данных по исследованию жидкой и твердой фаз нескольких электропроводных материалов.

Показано, что для определенных диапазонов температур, в основном выше обычных, справедливы простые корреляционные соотношения типа  $\lambda = LT\sigma + C$ , впервые предложенные Смитом и Полмером [1]. При соответствующем подборе постоянных  $L$  и  $C$  (по группам металлов и сплавов) величину  $\lambda$  можно рассчитать с точностью до 5–10%—порядок точности, достаточный для многих практических целей. Для таких материалов как бериллий, хром и графит, у которых теплопроводность фононами велика, требуется специальная обработка. Представляется необходимым проведение дальнейших исследований этих металлов и сплавов, а также сплавов вольфрама с кобальтом и расплавленной меди. Необходимо провести измерения для расплавленной меди, поскольку полученные до сих пор для нее значения располагаются 13–37% ниже прямой  $\lambda = 2,32 \times 10^{-8} \sigma T + 0,012$ , которая с точностью до 6% описывает представленные в настоящей работе данные для алюминия и данные, имеющиеся для нескольких расплавленных металлов и сплавов с меньшими значениями  $\lambda$ .